

Hawking radiation in GHS and non-extremal D1-D5 blackhole via covariant anomalies

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Abstract

We apply the method of Banerjee and Kulkarni (arXiv:0707.2449, [hep-th]) to provide a derivation of Hawking radiation from the GHS (stringy) blackhole which falls in the class of the most general spherically symmetric blackholes ($\sqrt{-g} \neq 1$) and also the non-extremal D1 – D5 blackhole using only covariant gravitational anomalies.

Keywords: Hawking radiation, anomaly

Introduction :

Hawking radiation is one of the most prominent quantum effect that arises for quantum fields in a background spacetime with an event horizon. The radiation is found to have a spectrum with Planck distribution giving the blackholes one of its thermodynamic properties. There are several derivations and all of them take the quantum effect of fields in blackhole backgrounds into account in various ways. The original derivation by Hawking [1] [2] calculates the Bogoliubov coefficients between the in and out states of fields in a blackhole background. A tunneling picture [3, 4] is based on pair creations of particles and antiparticles near the horizon and calculates WKB amplitudes for classically forbidden paths. A common property in these derivations is the universality of the radiation: i.e. Hawking radiation is determined universally by the horizon properties (if we neglect the grey body factor induced by the effect of scattering outside the horizon).

Another approach to the Hawking radiation is to calculate the energy-momentum (EM) tensor in the blackhole backgrounds. Classically, the EM tensor of any field is expected to be covariantly conserved in a curved background. However, quantum mechanically this is not always the case. For example, for a chiral scalar field in $(1+1)$ -dimensional curved spacetime the covariant derivative of the EM tensor reads

$$\nabla_\mu T^\mu{}_\nu = \frac{1}{96\pi\sqrt{-g}} \epsilon^{\beta\delta} \partial_\delta \partial_\alpha \Gamma^\alpha{}_{\nu\beta} \quad (1)$$

the right hand side being the consistent gravitational anomaly in that spacetime ([5, 6, 7, 8]). Under certain simplifying assumptions, it was shown by Christensen and Fulling [9], that the above anomaly can be interpreted as a flux of radiation, which quantitatively agrees with the Hawking flux [1, 2], from a horizon in that spacetime.

Recently, the above idea was resurrected by Robinson and Wilczek who showed (without many of the previous assumptions) that the above result was valid for a variety of spacetimes ([10]). The method was soon extended to the case of charged blackholes [11]. Further applications of this approach may be found in [12]-[21]. The basic idea in [10, 11] is that the effective theory near the horizon becomes two-dimensional and chiral. This chiral theory is anomalous. Using the form for two dimensional consistent gauge/gravitational anomaly Hawking fluxes are obtained. However the boundary condition necessary to fix the parameters are obtained from a vanishing of covariant current and energy-momentum tensor at the horizon. A more conceptually cleaner and economical approach based on cancellation of covariant (gauge/gravitational) anomaly has been discussed in [22]. Since the boundary condition involved the

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vanishing of covariant current/energy-momentum tensor at the horizon, it is more natural to make use of covariant expressions for gauge and gravitational anomaly. The generalization of this approach to higher spin field has been done in [23]. The spacetimes considered in this method included many of the known spherically symmetric spacetimes. Also, we would like to point out that an alternative derivation of Hawking flux based on effective action using only covariant anomaly has been discussed in [24].

In this paper, we adopt the method in ([22]) to discuss Hawking radiation from blackhole backgrounds in string theory. First we discuss the Garfinkle-Horowitz-Strominger (GHS) blackhole which is an example of the most general spherically symmetric blackhole spacetime ($\sqrt{-g} \neq 1$) [25, 26] and then we study the non-extremal D1-D5 blackhole [30].

Hawking radiation from GHS blackhole :

The GHS blackhole is a member of a family of solutions to low-energy string theory described by the action (in the string frame)

$$\Gamma = \int d^4x \sqrt{-g} e^{-2\phi} [-R - 4(\nabla\phi)^2 + F^2] \quad (2)$$

where ϕ is the dilaton field and $F_{\mu\nu}$ is the Maxwell field associated with a $U(1)$ subgroup of $E_8 \times E_8$ or $Spin(32)/Z_2$. Its charged black hole solution is given by

$$ds_{string}^2 = -f(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2 d\Omega \quad (3)$$

where,

$$\begin{aligned} f(r) &= \left(1 - \frac{2Me^{\phi_0}}{r}\right) \left(1 - \frac{Q^2 e^{3\phi_0}}{Mr}\right)^{-1} \\ h(r) &= \left(1 - \frac{2Me^{\phi_0}}{r}\right) \left(1 - \frac{Q^2 e^{3\phi_0}}{Mr}\right) \end{aligned} \quad (4)$$

with ϕ_0 being the asymptotic constant value of the dilaton field. We consider the case when $Q^2 < 2e^{-2\phi_0}M^2$ for which the above metric describes a blackhole with an event horizon situated at

$$r_H = 2Me^{\phi_0} . \quad (5)$$

With the aid of dimensional reduction procedure one can effectively describe a theory with a metric given by the by the “ $r-t$ ” sector of the full spacetime metric (3) near the horizon.

Now we divide the spacetime into two regions. In the region outside the horizon the theory is free from anomaly and hence we have the energy-momentum tensor satisfying the conservation law

$$\nabla_\mu T_{(o)\nu}^\mu = 0 . \quad (6)$$

However, the omission of the ingoing modes in the region $r \in [r_+, \infty]$ near the horizon, leads to an anomaly in the energy-momentum tensor there. As we have mentioned earlier, in this paper we shall focus only on the covariant form of $d=2$ gravitational anomaly given by ([10, 11]):

$$\nabla_\mu T_{(H)\nu}^\mu = \frac{1}{96\pi} \bar{\epsilon}_{\nu\mu} \partial^\mu R = \mathcal{A}_\nu \quad (7)$$

where, $\bar{\epsilon}^{\mu\nu} = \epsilon^{\mu\nu}/\sqrt{-g}$ and $\bar{\epsilon}_{\mu\nu} = \sqrt{-g}\epsilon_{\mu\nu}$ are two dimensional antisymmetric tensors for the upper and lower cases with $\epsilon^{tr} = \epsilon_{rt} = 1$. It is easy to check that for the metric (3), the anomaly is purely timelike with

$$\begin{aligned} \mathcal{A}_r &= 0 \\ \mathcal{A}_t &= \frac{1}{\sqrt{-g}} \partial_r N_t^r \end{aligned} \quad (8)$$

where,

$$N_t^r = \frac{1}{96\pi} \left(h f'' + \frac{f' h'}{2} - \frac{f'^2 h}{f} \right) . \quad (9)$$

Now outside the horizon, the conservation equation (6) yields the differential equation

$$\partial_r(\sqrt{-g}T_{(o)t}^r) = 0 \quad (10)$$

which after integration leads to

$$T_{(o)t}^r(r) = \frac{a_o}{\sqrt{-g}} \quad (11)$$

where, a_o is an integration constant. In the region near the horizon, the anomaly equation (7) leads to the following differential equation

$$\partial_r(\sqrt{-g}T_{(H)t}^r) = \partial_r N_t^r(r) \quad (12)$$

which after solution leads to

$$T_{(H)t}^r = \frac{1}{\sqrt{-g}}(b_H + N_t^r(r) - N_t^r(r_H)) \quad (13)$$

where, b_H is an integration constant.

Now as in ([11], [22]), writing the energy-momentum tensor as a sum of two contributions

$$T_t^r(r) = T_{(o)t}^r(r)\theta(r - r_H - \epsilon) + T_{(H)t}^r(r)H(r) \quad (14)$$

where, $H(r) = 1 - \theta(r - r_H - \epsilon)$, we find

$$\begin{aligned} \nabla_\mu T^\mu_t &= \partial_r T_t^r(r) + \partial_r(\ln \sqrt{-g})T_t^r(r) \\ &= \frac{1}{\sqrt{-g}}\partial_r(\sqrt{-g}T_t^r(r)) \\ &= \frac{1}{\sqrt{-g}}\left[\left(\sqrt{-g}(T_{(o)t}^r(r) - T_{(H)t}^r(r)) + N_t^r(r)\right)\delta(r - r_H - \epsilon) + \partial_r(N_t^r(r)H(r))\right]. \end{aligned} \quad (15)$$

The term in the total derivative is cancelled by quantum effects of classically irrelevant ingoing modes. The quantum effect to cancel this term is the Wess-Zumino term induced by the ingoing modes near the horizon. Hence the vanishing of the Ward identity under diffeomorphism transformation implies that the coefficient of the delta function in the above equation vanishes

$$T_{(o)t}^r - T_{(H)t}^r + \frac{N_t^r(r)}{\sqrt{-g}} = 0. \quad (16)$$

Substituting (11) and (13) in the above equation, we get

$$a_o = b_H - N_t^r(r_H). \quad (17)$$

The integration constant b_H can be fixed by imposing that the covariant energy-momentum tensor vanishes at the horizon. From (13), this gives $b_H = 0$. Hence the total flux of the energy-momentum tensor is given by

$$\begin{aligned} a_o &= -N_t^r(r_H) \\ &= \frac{1}{192\pi}f'(r_H)g'(r_H). \end{aligned} \quad (18)$$

Using (4), we finally obtain

$$a_o = \frac{\pi}{12}T_H^2 \quad (19)$$

where T_H is the Hawking temperature given by

$$T_H = \frac{1}{8\pi M e^{\phi_0}}. \quad (20)$$

This is precisely the Hawking flux obtained in ([27]) using Robinson-Wilczek method of cancellation of consistent anomaly.

Finally at extremality, i.e. when $Q^2 = 2e^{-2\phi_0} M^2$, the GHS blackhole solution (3, 4) becomes

$$ds^2 = -dt^2 + \left(1 - \frac{2Me^{\phi_0}}{r}\right)^{-2} + r^2 d\Omega . \quad (21)$$

It is easy to check that in this case the Hawking temperature vanishes. Indeed, explicit computation of N_t^r for the above metric (21) shows that the energy flux vanishes.

Hawking radiation from D1-D5 non-extremal blackhole :

As another example of covariant anomaly cancellation approach, we consider a non-extremal five dimensional blackhole which originates as a brane configuration in Type IIB superstring theory compactified on $S^1 \times T^4$. The configuration relevant to the present case is composed of D1-branes wrapping S^1 , D5-branes wrapping $S^1 \times T^4$ and momentum modes along S^1 . The solution of the Type IIB supergravity corresponding to this configuration is a supersymmetric background known as the extremal five-dimensional D1-D5 blackhole having zero Hawking temperature. Hence in order to consider Hawking radiation we study the non-extremal D1-D5 blackhole.

The ten-dimensional supergravity background corresponding to the non-extremal D1-D5 blackhole has the following form in the string frame [28]:

$$\begin{aligned} ds_{10}^2 &= f_1^{-1/2} f_5^{-1/2} (-h f_n^{-1} dt^2 + f_n (dx_5 + (1 - \tilde{f}_n^{-1}) dt)^2) \\ &\quad + f_1^{1/2} f_5^{-1/2} (dx_6^2 + \dots + dx_9^2) + f_1^{1/2} f_5^{1/2} (h^{-1} dr^2 + r^2 d\Omega_3^2) \\ e^{-2\phi} &= f_1^{-1} f_5 \quad , \quad C_{05} = \tilde{f}_1^{-1} - 1 \\ F_{ijk} &= \frac{1}{2} \epsilon_{ijkl} \partial_l \tilde{f}_5 \quad , \quad i, j, k, l = 1, 2, 3, 4 \end{aligned} \quad (22)$$

where, x_5 and x_6, \dots, x_9 are periodic coordinates along S^1 and T^4 respectively and F is the three-form field strength of the RR 2-form gauge potential C , $F = dC$. Also various functions appearing in the above background are functions of coordinates x_1, \dots, x_4 given by

$$\begin{aligned} h &= 1 - \frac{r_0^2}{r^2} \quad , \quad f_{1,5,n} = 1 + \frac{r_{1,5,n}^2}{r^2} \\ \tilde{f}_{1,n}^{-1} &= 1 - \frac{r_0^2 \sinh \alpha_{1,n} \cosh \alpha_{1,n}}{r^2} f_{1,n}^{-1} \\ r_{1,5,n}^2 &= r_0^2 \sinh^2 \alpha_{1,5,n} \quad , \quad r^2 = x_1^2 + \dots + x_4^2 \end{aligned} \quad (23)$$

where, r_0 is the extremality parameter and $h, f_{1,5,n}$, are harmonic functions representing the non-extremality and the presence of D1, D5, and momentum modes respectively.

Dimensional reduction of (22) along $S^1 \times T^4$ following the procedure of [29] yields the Einstein metric of the non-extremal five-dimensional blackhole as

$$ds_5^2 = -\lambda^{-2/3} h dt^2 + \lambda^{1/3} (h^{-1} dr^2 + r^2 d\Omega_3^2) \quad (24)$$

where λ is defined by

$$\lambda = f_1 f_5 f_n . \quad (25)$$

The event horizon r_H of this blackhole geometry is located at

$$r_H = r_0 . \quad (26)$$

Apart from the metric, the dimensional reduction gives us three kinds of gauge fields. The first one is the Kaluza-Klein gauge field $A_\mu^{(K)}$ coming from the metric and the second one, say $A_\mu^{(1)}$, basically stems from $C_{\mu 5}$. (We note that $\mu = 0, 1, 2, 3, 4$.) From the background (22), the first two gauge fields are obtained as

$$A^{(K)} = -(\tilde{f}_n^{-1} - 1) dt \quad , \quad A^{(1)} = (\tilde{f}_1^{-1} - 1) dt . \quad (27)$$

Unlike these gauge fields which are one-form in nature, the last one is the two-form gauge field $A_{\mu\nu}$, originating from $C_{\mu\nu}$ whose field strength is given by the expression of F in (22).

Now if we consider a free complex scalar field in the black hole background (24) and (27) and perform a partial wave decomposition of φ in terms of the spherical harmonics, then it can be shown that the action near the horizon becomes [30]

$$S[\varphi] = - \sum_a \int dt dr r^3 \lambda^{1/2} \varphi_a^*(t, r) \left(-\frac{1}{f} (\partial_t - iA_t)^2 + \partial_r f \partial_r \right) \varphi_a(t, r) \quad (28)$$

where, $A_t = e_1 A_t^{(1)} + e_K A_t^{(K)}$ and a is the collection of angular quantum numbers of the spherical harmonics. It can be easily checked that this action describes an infinite set of massless two-dimensional complex scalar fields in the following background :

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 \quad , \quad \Phi = r^3 \lambda^{1/2} \quad (29)$$

$$A_t(r) = -\frac{e_1 r_o^2 \sinh \alpha_1 \cosh \alpha_1}{r^2 + r_o^2} + \frac{e_K r_n^2 \sinh \alpha_n \cosh \alpha_n}{r^2 + r_n^2} \quad (30)$$

where Φ is the two-dimensional dilaton field.

As we have stated earlier, the two dimensional effective theory near the horizon (28) possesses gravitational as well as gauge anomaly. We once again follow the approach based on covariant anomaly cancellation proposed in [22]. We first consider the gauge part. Since there are two kinds of $U(1)$ gauge symmetries, we have two $U(1)$ gauge currents $J_\mu^{(1)}$ and $J_\mu^{(K)}$ corresponding to $A_\mu^{(1)}$ and $A_\mu^{(K)}$ respectively. The form for covariant gauge anomaly for these two currents are identical in nature, therefore we discuss the case for $J_\mu^{(1)}$ explicitly and just mention the result for the other. Since the spacetime has been divided into two regions, we divide the current $J^{(1)\mu}$ into two parts. The current outside the horizon denoted by $J_{(o)}^{(1)\mu}$ is anomaly free and hence satisfies the conservation law

$$\nabla_\mu J_{(o)}^{(1)\mu} = 0 \quad (31)$$

while the current near the horizon satisfies

$$\nabla_\mu J_{(H)}^{(1)\mu} = -\frac{e_1}{4\pi} \bar{\epsilon}^{\rho\sigma} F_{\rho\sigma} = \frac{e_1}{2\pi} \partial_r A_t . \quad (32)$$

Solving (31) and (32) in the region outside and near the horizon, we get

$$J_{(o)}^{(1)r} = c_o^{(1)} \quad (33)$$

$$J_{(H)}^{(1)r} = c_H^{(1)} + \frac{e_1}{2\pi} [A_t(r) - A_t(r_H)] . \quad (34)$$

Now as in [11], writing $J^{(1)r}$ as

$$J^{(1)r} = J_{(o)}^{(1)r} \Theta(r - r_H - \epsilon) + J_{(H)}^{(1)r} H(r) \quad (35)$$

we find

$$\nabla_\mu J^{(1)\mu} = \partial_r J^{(1)r} = \partial_r \left(\frac{e_1}{2\pi} A_t H \right) + \delta(r - r_H - \epsilon) \left[J_{(o)}^{(1)r} - J_{(H)}^{(1)r} + \frac{e_1}{2\pi} A_t \right] . \quad (36)$$

Now the vanishing of the Ward identity under gauge transformation requires that the first term must be cancelled by quantum effects of classically irrelevant ingoing modes. The vanishing of the second term implies that the coefficient of the delta function is zero, leading to the condition

$$c_o^{(1)} = c_H^{(1)} - \frac{e_1}{2\pi} A_t(r_H) . \quad (37)$$

The coefficient $c_H^{(1)}$ vanishes by requiring that the covariant current $J_{(H)}^{(1)r}$ vanishes at the horizon. Hence the charge flux corresponding to $J^{(1)r}$ is given by

$$c_o^{(1)} = -\frac{e_1}{2\pi} A_t(r_H) = \frac{e_1}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) . \quad (38)$$

Following the same procedure for $J^{(K)\mu}$ satisfying the anomaly equation

$$\nabla_\mu J_{(H)}^{(K)\mu} = -\frac{e_K}{4\pi} \bar{\epsilon}^{\rho\sigma} F_{\rho\sigma} = \frac{e_K}{2\pi} \partial_r A_t \quad (39)$$

we find that the charge flux corresponding to current $J^{(K)r}$ reads

$$c_o^{(K)} = -\frac{e_K}{2\pi} A_t(r_H) = \frac{e_K}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) . \quad (40)$$

Hence the total charge flux is given by

$$c_o = c^{(1)} + c_o^{(K)} = -\frac{e}{2\pi} A_t(r_H) = \frac{e}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) . \quad (41)$$

where, $e = e_1 + e_K$.

Now we move on to the problem of computing the energy flux. Since we have an external gauge field, the energy-momentum tensor will not satisfy the conservation law even at classical level, rather it gives rise to the Lorentz force law, $\nabla_\mu T^\mu{}_\nu = F_{\mu\nu} J^\mu$ ¹. Hence the corresponding expression for the anomalous Ward identity for covariantly regularised quantities is given by [22]

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu} J^\mu + \mathcal{A}_\nu \quad (42)$$

where, \mathcal{A}_ν is the two-dimensional gravitational covariant anomaly (7). In the region outside the horizon, there is no anomaly and hence the Ward identity reads

$$\nabla_\mu T^\mu{}_{(o)\nu} = \partial_r T^r{}_{(o)t} = F_{rt} J_{(o)}^r \quad (43)$$

Using (33), the above equation can be solved as

$$T_{(o)t}^r(r) = a_o + c_o A_t(r) \quad (44)$$

where, a_o is an integration constant. However near the horizon the Ward identity reads

$$\partial_r T_{(H)t}^r = F_{rt} J_{(H)}^r + \partial_r N_t^r \quad (45)$$

where, N_t^r is given by (9) with $h(r) = f(r)$. Now substituting $J_{(H)}^r = J_{(H)}^{(1)r} + J_{(H)}^{(K)r}$, we get

$$T_{t(H)}^r = a_H + \int_{r_H}^r dr \partial_r \left[c_o A_t + \frac{e}{4\pi} A_t^2 + N_t^r \right] . \quad (46)$$

Now following the same procedure as given in the gauge part we arrive at the relation

$$a_o = a_H + \frac{e}{4\pi} A_t^2(r_H) - N_t^r(r_H) . \quad (47)$$

Implimenting the boundary condition that covariant energy momentum tensor vanishes at the horizon as before fixes a_H to be zero. Therefore, a_o is given by

$$a_{(o)} = \frac{e}{4\pi} A_t^2(r_H) - N_t^r(r_H) . \quad (48)$$

Now substituting the values for A_t and N_t^r at the horizon, we get

$$\begin{aligned} a_o &= \frac{e}{4\pi} A_t^2(r_H) + N_t^r(r_+) \\ &= \frac{e}{4\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n)^2 + \frac{\pi}{12} T_H^2 \end{aligned} \quad (49)$$

where,

$$T_H = \frac{1}{2\pi r_0 \cosh \alpha_1 \cosh \alpha_5 \cosh \alpha_n} . \quad (50)$$

¹Note that here $J^\mu = J^{(1)\mu} + J^{(K)\mu}$.

This is just the energy flux from blackbody radiation with two chemical potentials for the charges e_1 and e_K [30].

Discussions :

In this paper, we studied the problem of Hawking radiation from blackhole spacetimes that occur in string theory using covariant anomaly cancellation technique proposed in [22]. The point is that Hawking radiation plays the role of cancelling gauge and gravitational anomalies at the horizon to restore the gauge/diffeomorphism symmetry at the horizon. An advantage of this method is that neither the consistent anomaly nor the counterterm relating the different (covariant and consistent) currents, which were essential ingredients in [11, 27, 30], were required.

We discussed in particular Hawking radiation from GHS and five dimensional non-extremal D1-D5 blackhole. For GHS blackhole, the energy-momentum flux was obtained when $Q^2 < 2Me^{-2\phi_0}$. At extremality there is no energy flux and hence Hawking temperature is zero. In the case of $D1 - D5$ blackhole, fluxes of electric charge flow and energy-momentum tensor were obtained. The resulting fluxes are the same as that of the two dimensional black body radiation at the Hawking temperature.

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